PROCEEDINGS OF THE SAWTOOTH SOFTWARE CONFERENCE

November 2022

VOLUMETRIC CONJOINT AND THE ROLE OF ASSORTMENT SIZE

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ABSTRACT

Volume predictions based on conjoint analysis are particularly challenging in packaged goods categories where variety seeking is common, and consumers simultaneously buy multiple brands. Extant volumetric demand models applied to volumetric choice experiments are unable to deal with variation in assortment size. We extend Multiple Discrete Continuous Models to include a relationship between assortment size and marginal utilities. Using two volumetric conjoint studies in different categories (chocolate bars and air fresheners), we demonstrate the proposed model's ability to predict demand for market-like scenarios, while analogous MDCMs over-predict primary demand by 40%–80%.

1. INTRODUCTION

Conjoint analysis is often used as a basis for sales volume predictions. Using choice shares to predict volumes is straightforward when several assumptions are met, including a fixed market size and independence of preferences and quantity demanded. These assumptions may not be appropriate in packaged goods where many consumers buy more than one brand simultaneously, and some brands might be more popular for consumers who buy larger quantities, while other brands or features are more popular with those who tend to buy lower quantities. Extant volumetric demand models are able to describe simultaneous demand of multiple varieties and capture preference-quantity relationships, but we show that these models are inappropriate when there is variation in assortment size or choice set size. However, choice set sizes in conjoint are usually much smaller than assortments in store, rendering extant demand models useless for most volume prediction tasks.

Assortment size variation also matters for retailers trying to optimize their assortments with respect to their composition and size. The introduction of a new line of store brand products might grow the category altogether or just "steal" shares from other brands. In model terms, the outcome depends on consumer budgets, preferences, satiation and the number of choice alternatives for each scenario. Extant models are too inflexible to deal with variation of assortment size.

We build on Multiple Discrete Continuous Models (MDCMs) that are often used to model volumetric demand, and propose a parameterization of assortment size that captures negative effects of choice-set size on inside good marginal utilities (which is equivalent to positive effects on the marginal utility of the outside good). While demand for inside goods will often increase

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with growing assortment size, the marginal utility of an individual unit can decrease. Once a certain assortment size is reached, additional increases in size might even result in decreasing demand. The model is thus able to describe phenomena described by behavioral researchers. For instance, Dhar (1997) finds that larger assortments may lead to deferral of choice. Schwartz (2016) suggests that "more is less." We do not expect to find "choice overload" in our typical packaged goods applications, but the model would be able to describe patterns consistent with that idea.

To understand why extant MDCM models are inappropriate when assortment size varies, we need to review how these models work. They allow for "corner solutions" (products that are not purchased) and multiple "interior solutions" (products that are bought, where the purchase quantity is continuous). Multiple interior solutions are possible because it assumed that there are diminishing marginal utilities to all goods. Consuming only the good with the highest baseline marginal utility might not be utility-optimal, since the marginal increase in utility decreases. Instead, consumers may choose to buy several different goods at the same time. When several additional product varieties are added to the choice-set, more products can be purchased at a given marginal rate of utility, resulting in increased overall demand for inside goods. It is therefore built into these multiple discrete-continuous models that primary demand is monotonically increasing with choice-set size. The relationship between set size and primary demand is governed by parameters already identified in the absence of set size variation. This means that these models are over-identified when choice-set size varies. These models are unable to explain negative relationships between inside good marginal utilities and assortment size and are thus prone to overpredict demand for scenarios with larger assortment sizes.

We demonstrate the performance of our model using two volumetric conjoint studies of chocolate bars in Germany and non-electric air fresheners in the US. Parameters governing secondary demand ("utilities" or "part-worths") are largely unaffected by set size variation, however, ignoring set size variation leads to dramatic over-prediction of primary demand. Extant models are off by as much as 80% in our first study and 40% in our second study.

2. Our Proposed Model

For an overview and the economic background of choice, see Allenby et al. (2019) and Dubé (2019). MDCMs can explain simultaneous demand for multiple distinct products. For each so-called interior solution (i.e., each product that is bought), demand quantities are assumed continuous. This simplifying assumption allows developing models based on the Karush-Kuhn-Tucker conditions which are computationally tractable.

The economic assumptions behind these models are straightforward: Decision makers maximize utility subject to a budget constraint. The utility maximization problem for a single choice occasion (i.e., a single choice task or shopping trip) can be expressed as:

$$\text{Max } u(\mathbf{x}, z) = \sum_{j=1}^{N} \frac{\psi_j}{\gamma} \ln\left(\gamma x_j + 1\right) + \psi_z \ln(z) \quad \text{s.t.} \quad \mathbf{p}'\mathbf{x} + z \leq E \tag{0.1}$$

Here, x_j is the purchased quantity of good j, and ψ_j represents the baseline preference for that good j. The rate of satiation of inside goods is controlled by γ, and p_j is the price of a unit of good j. The outside good z represents unspent money that the decision maker has been willing to allocate towards the focal category, but eventually did not end up spending on inside goods available in the choice set. We assume that there are diminishing returns to unspent money, and therefore use a nonlinear specification of z. This allows estimating the budgetary allotment E, which is identified through the functional form of the utility function. Baseline marginal utility of good j is defined as follows, assuming multiplicative, independent error terms for each of the inside goods:

(0.2) $$\psi_j = \exp\left(\mathbf{a}_j \boldsymbol{\beta} + \varepsilon_j\right)$$

where β is the vector of "part-worths" and $\mathbf{a}_j$ is the design vector for alternative j, and ε_j is a random term. $\mathbf{a}_j$ can be specified using dummy coding, in which case the first element of β, β_0, serves as an intercept capturing the baseline marginal utility of an inside good vs the outside good. Alternatively, effects coding can be used. The corresponding likelihood function can be developed by exploiting the Karush-Kuhn-Tucker (KKT) conditions. For the purpose of identification, and without loss of generality, it is common to constrain $\psi_z = 1$, which reduces the dimensionality of the system of equations defined by the KKT conditions by one.

This specification implies that, unless the budget constraint is binding, (1) primary demand is increasing in choice-set size, and (2) the strength of this relationship is determined by parameters which are already identified in the absence of set size variation. In other words, once set size variation introduced, the model is over-identified, and it is possible to identify a parameter in place of ψ_z if it is a function of N_t. A simple specification is shown in the equation below, where outside good baseline marginal utility ψ_z is a function of N_t and a set size parameter ξ:

(0.3) $$\psi_z = f\left(N_t;\xi\right) = \exp\left(0 + \ln(f\left(N_t;\xi\right))\right)$$

Here, ξ must be constrained to be positive, because stronger relationships between set size and primary demand can be represented by corresponding combinations of β_0, γ, E already. This constraint will ensure identification. Depending on the amount of information available, $f(N_t;\xi)$ can be specified in more or less flexible ways, or even be estimated non-parametrically. In our first empirical application, we only observe two discrete sizes of the choice set. In this case, a simple linear specification can be fit:

(0.4) $$f\left(N_t;\xi_1\right) = \xi_1 N_t + 1$$

This parameterization implies that the marginal utility of the outside good is increasing in N_t. Relatively speaking, the utility of each inside good is decreasing in N_t. The resulting model nests the extant volumetric demand model when $\xi = 0$. Larger values of ξ mean that consumers do attach higher relative marginal utility to the outside good as N_t increases. In our second empirical application, we observe three different choice-set sizes. In that case, we can also fit a 2nd order polynomial. The appropriate order of the polynomial can be identified by comparing models based on the log-marginal likelihood.

The model likelihood is straightforward to derive.

(0.5) $$\Pr(\mathbf{x}) = |J_R| \left\{ \prod_{j=1}^{R} \frac{\exp(-g_j/\sigma)}{\sigma} \right\} \exp\left\{ -\sum_{i=1}^{N} \exp(-g_i/\sigma) \right\}$$

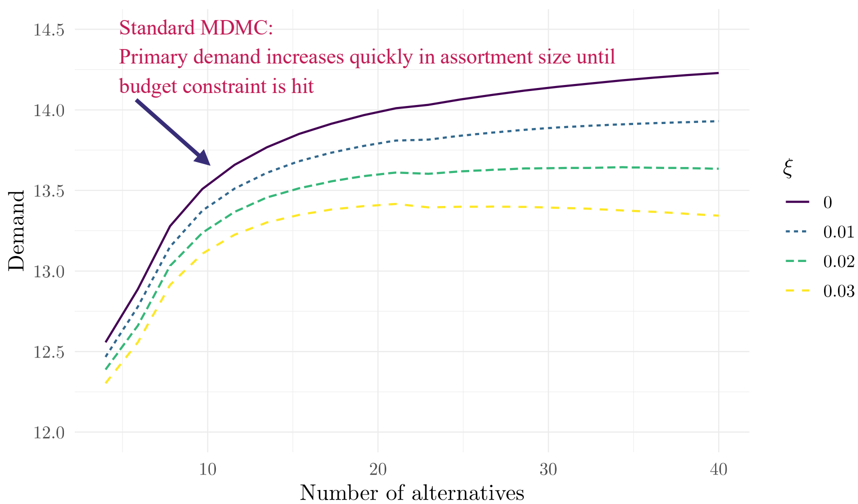
where $$g_{kt} = -\mathbf{a}_{kt}\beta + \ln\left(\xi N_t + 1\right) + \ln\left(p_{kt}\right) + \ln\left(\gamma x_{kt} + 1\right) - \ln\left(z_t\right)$$

and $$\left|J_R\right| = \prod_{j=1}^{R}\left(\frac{\gamma}{\gamma x_j + 1}\right)\left\{\sum_{j=1}^{R}\frac{\gamma x_j + 1}{\gamma}\cdot\frac{p_j}{z} + 1\right\}$$

It is important to remember that ξ is only identified when there is variation in N_t. The source of variation in N_t can be experimental (e.g., in a choice experiment) or natural (when a store changes assortments over time in purchase transaction or similar “revealed preference” data).

To illustrate the influence of ξ on primary demand, we use a simulation exercise. We compute expected demand for a single decision maker, varying N_t and ξ, while all choice alternatives have the same deterministic utility. The resulting primary demand curves are shown in Figure 1. $\xi = 0$ corresponds to the simple volumetric demand model, while larger values of ξ show smaller increases in primary demand, or even decreasing primary demand. Comparing the different demand curves, we see that significant variation in the number of alternatives may be necessary to notice the relationship. An increase from 8 to 12 choice alternatives may only have a small impact on primary demand. However, once we consider demand in much larger assortments with 20 or more alternatives, there are considerable differences in predicted primary demand.

Figure 7: Assortment Size and Primary Demand



2.1 Heterogeneity and Estimation

$\theta_h = \{\beta_h, \ln\gamma_h, \ln E_h, \ln\sigma_h, \ln\xi_h\}$ is subject/respondent h’s vector of parameters of length M governing the individual-level demand model. We assume a simple Multivariate Normal model of heterogeneity, i.e., $\theta_h \sim \text{Normal}(\bar{\theta}, \Sigma)$.

2.2 Demand Predictions

Demand x_{ht} from consumer h at time t is a function of parameters of the demand model (θ_h), a realization of the vector of error terms (ε_{ht}), and characteristics of the available assortment including prices. We call the demand function D:

$$\mathbf{x}_{ht} = \mathrm{D}\left(\theta_h, \boldsymbol{\varepsilon}_{ht} \mid \mathbf{A}_t, \mathbf{p}_t\right) \tag{0.6}$$

There is no closed form solution for it. However, it can be computed using an iterative procedure that at worst takes R_t iterations. Finally, expected demand is obtained by integrating out the error term and posterior distribution of model parameters θ_h. Numeric integration is computationally cheap, because draws of θ have already been produced in the process of estimating the model, and D can easily be computed.

3. Empirical Application

We use data from two studies to investigate the properties of our proposed demand model. Both datasets are collected from commercials panels. In both studies, we use experimental choice-set size variation which can help identify the proposed set size parameter(s). We use the estimated models to extrapolate from the relative small-N experimental world to market-like large-N scenarios. The following models will be applied:

vd an extant specification of a **v**olumetric demand model

vd-ss(o) our proposed model (where o is the order of the polynomial)

To identify respondents with unrealistic or incoherent preferences, we first estimate simple volumetric demand models (**vd**) for each set-size, obtain individual log-likelihood values and remove about the 10% of worst-fitting respondents. We also remove respondents who never choose a single choice alternative.

3.1 Chocolate Bars

The design of the German chocolate bar volumetric conjoint follows standard procedures, except for the presence choice-set size variation (8 and 18 alternatives per task). In order to test how accurately competing demand models predict market-level demand in a "base case" scenario (i.e., current market demand at current market offerings), the study is designed to reproduce a set of typical market offerings available in supermarkets across Germany in 2018. Therefore, no new flavors or flavor combinations were added to the study.

Table 3: Attributes and Levels (Chocolate Bars)

Attributes	Levels
Brand	Alpia, Feodora, Kinder, Lindt, Merci, Milka, Nestle, Ritter, Sarotti, Schogetten, Suchard, Tobler, Trumpf, Ferrero/Yogurette
Chocolate	Milk, Dark, Black, White
Nut	Nut, No Nut
Fruit	Fruit, Berry, Grape, No Fruit
Filling	None, Yogurt, Choc Chunk, Coffee, Cookie, Black and White, Crisp, Nougat, Caramel, Milk Creme, Special, Marzipan

We characterized chocolate bars in terms of five key attributes: Brand name, Chocolate type, Nut content, Fruit or Berry content and Filling. An overview of attributes and levels in shown in Table 1. Using those attributes and levels we can map between product space (with about 100 unique products accounting for 80% of sales volume) and the lower-dimensional attribute space. Figures 2 and 3 show example choice tasks with 8 and 18 alternatives, respectively.

Figure 8: 8 Alternatives (Chocolate Bars)

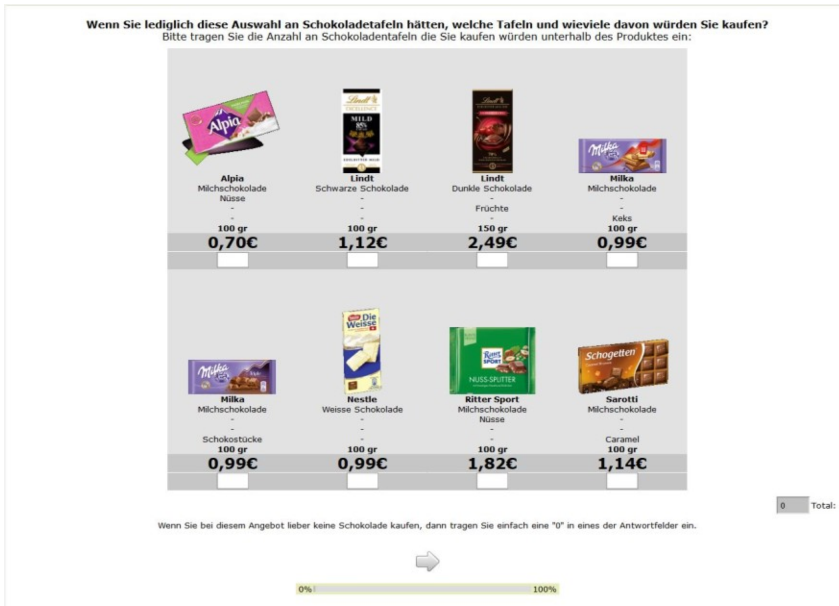
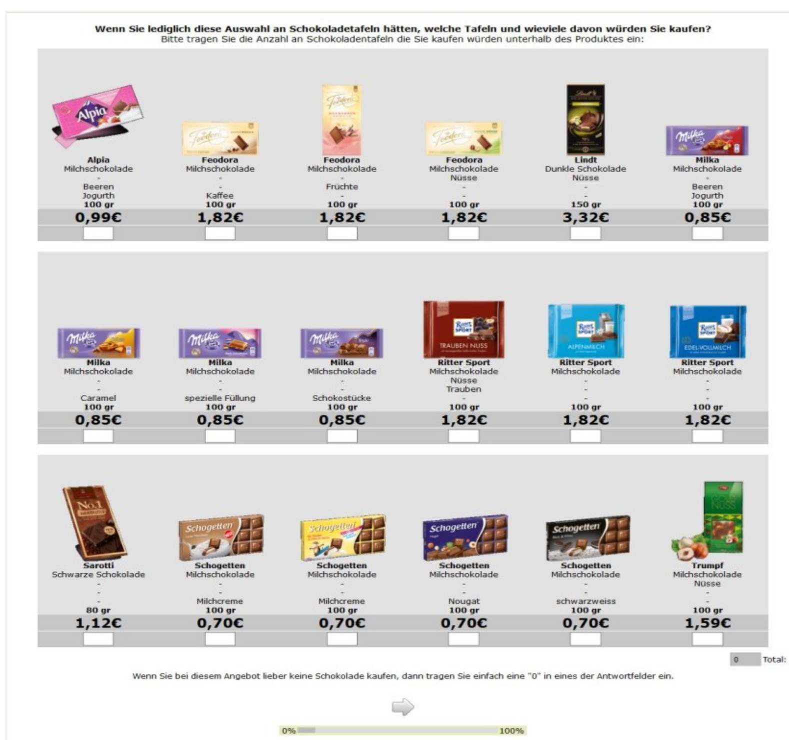


Figure 9: 18 Alternatives (Chocolate Bars)



Descriptive statistics of demand are summarized in Table 2. Respondents choose larger quantities (1.95 instead of 1.31) and more varieties (around 1.54 instead of 1.05) when offered a larger assortment. Therefore, they overall spend more when larger assortments are offered.

Table 4: Descriptive Statistics (Chocolate Bars)

Number of Alternatives	Units per task		Varieties per task		$ spent per task		Maximum spent	
	mean	sd	mean	Sd	mean	sd	mean	sd
8	**1.31**	1.25	**1.05**	0.93	1.53	1.55	3.20	1.91
18	**1.95**	2.08	**1.54**	1.61	2.29	2.56	4.44	3.27

We randomly select 1 choice task per respondent for out-of-sample fit statistic computation and estimate the proposed and benchmark models (vd-ss(1) and vd).

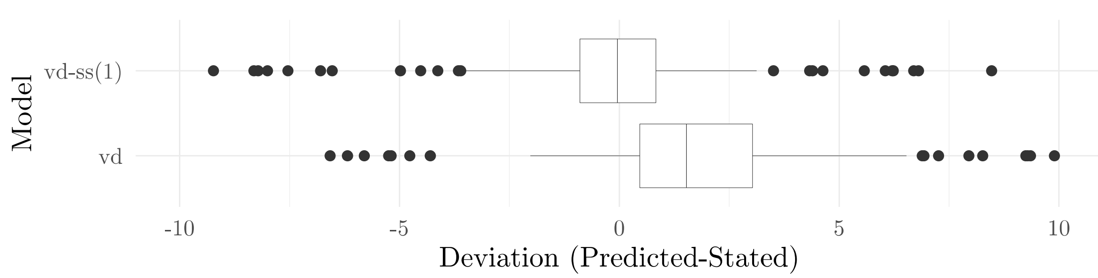
Table 5: Comparing Fit (Chocolate Bars)

	In-sample	Out of sample	
Model	LMD	MSE	MAE
Vd	-12,423	0.445	0.183
vd-ss(1)	-12,346	0.455	0.182

Fit statistics are presented in Table 3. It shows the log marginal density of the data (LMD) for in-sample fit, and the mean squared error (MSE) and mean absolute error (MAE) for out-of-sample fit. There are no dramatic differences in model fit between the models. This is to be expected, because choice-set size variation is limited to 8 and 18 alternatives. A much better test of external validity is based on the ability of the model to predict actual purchase behavior, beyond the respective choice experiment.

We use a "base case" scenario that mimics an assortment available at a typical German supermarket in 2018. It consists of 117 products, including their configuration and typical price. Focusing on primary (i.e., total) demand per respondent, we compare self-stated purchase quantity during the last shopping trip to predicted purchase quantity. Figure 4 shows distributions of absolute error for the proposed and competing models. It is clear that the extant model is biased, over-predicting self-reported quantities.

Figure 10: Predicted vs Self-Stated Quantity (Chocolate Bars)



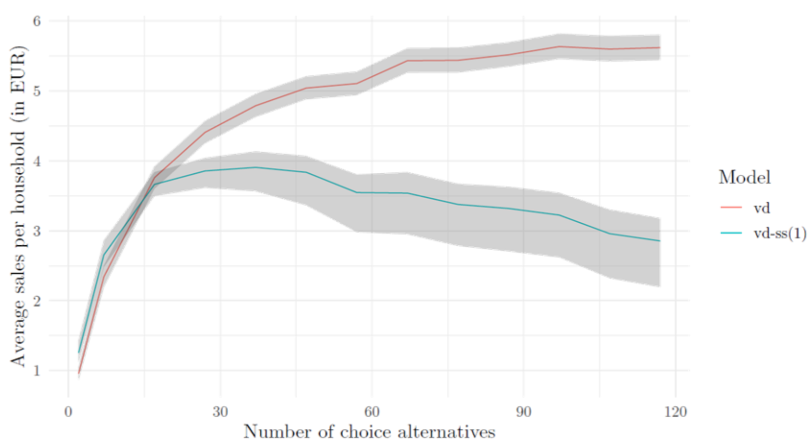
In order to project marketplace demand, we need to make additional assumptions: The number of households in Germany that regularly shop chocolate bars is around 25,000,000. Germans shop for chocolate almost every week, for an average of 3 shopping trips per month during which they shop for chocolate bars. These are simplifying assumptions, ignoring both purchase dynamics (e.g., stockpiling) and consumption dynamics (e.g., consumers eating more because "it is there"). Extrapolated marketplace demand estimates (in tons of chocolate) are shown in Table 4. For reference, we add an extrapolation based on stated quantity. From aggregate reports, we found that actual marketplace demand equals about 240,000t[3]. The extant model dramatically over-predicts demand, while the proposed model produces a realistic prediction.

Our model also allows counterfactuals with respect to assortment size. Figure 5 shows that about 20 offerings are sufficient to reach a high level of primary demand.

Table 6: Market Extrapolation (Chocolate Bars)

Model	E(demand)	CI-5%	CI-95%
Vd	434,730	419,298	450,810
vd-ss(1)	218,742	191,678	242,982
Based on stated quantity	215,422		
Actual	~240,000		

Figure 11: Counterfactualizing Assortment Size



3.2 Air Fresheners

In our second application, we conducted a volumetric choice experiment in the air "NECA" freshener category. These are simple non-electric air fresheners available in regular retail stores. Respondents were recruited from a commercial panel in the United States. They were shown 8, 16 and 24 choice alternatives for a total of 15 choice tasks. An example choice task with 16 alternatives is shown in Figure 6.

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[3] The latest actual marketplace demand number we found is from 2016. We have not seen evidence for dramatic changes in primary demand.

In order to assess the ability to extrapolate demand to scenarios with more choice alternatives, we showed respondents an initial “shelf task” with 57 choice alternatives. The assortment of 57 alternatives closely resembles a typical offering in a store. Moreover, this shelf task does not show an attribute grid, since it’s meant to best mimic a real-world purchase decision in a store.

Descriptive statistics of demand are summarized in Table 5. Summaries are broken down by number of choice alternatives shown. Primary demand increases as the set size is increased beyond 16 alternatives. This supports the general idea that consumers respond to increased variety by increasing primary demand.

Figure 12: Choice Task (Air Fresheners)

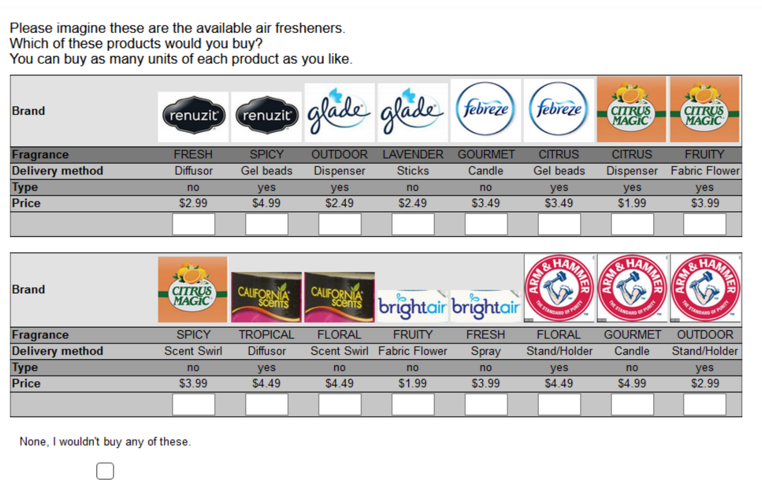

Table 7: Descriptive Statistics (Air Fresheners)

Number of Alternatives	Units per task		Varieties per task		$ spent per task		Maximum spent	
	mean	sd	mean	Sd	mean	sd	mean	sd
8	**1.05**	1.27	0.79	0.80	2.80	3.84	6.30	5.46
16	**1.04**	1.31	0.80	0.87	2.84	4.07	6.20	5.59
24	**1.43**	1.76	1.11	1.17	3.88	5.27	7.59	7.17
57	**4.11**	6.18	2.58	2.87	6.21	10.10	6.21	10.10

Table 6 shows the predictive accuracy of the competing models. Models with set size adjustment again outperform the extant model. Our proposed vd-ss(1) model produced the best in-sample fit and is able to generate more accurate predictions, with relative bias close to 0.

Table 8: Validation Task Fit (Air Fresheners)

Model	MSE	MAE	Bias
vd	1.19	0.17	0.03
vd-ss(1)	0.94	0.14	0.00
vd-ss(2)	0.94	0.14	0.00

For Table 7, we aggregated demand for the 57 products to the brand-name level to facilitate comparisons. The proposed vd-ss(1) model predict overall demand of 2,196 units from our 516 respondents. Actual demand in the shelf task was 2,120. The extant model predicts a demand of 2,953 units, over-predicting demand by almost 40%.

Table 9: Brand-Level Demand Predictions (Air Fresheners)

Brand	*Actual*	vd	**vd-ss(1)**	vd-ss(2)
Renuzit	1,137	1,675	1,256	1,225
Glade	479	750	559	555
Febreze	311	245	177	174
BrightAir	63	39	28	29
CitrusMagic	51	105	77	77
ArmHammer	40	14	10	10
CaliforniaScent	39	126	88	89
Total	2,120	2,953	2,196	2,162
Relative		**139%**	**104%**	**102%**

4. Summary and Conclusion

Volumetric conjoint analysis with set size variation is a great tool for policy simulations that involve the addition or removal of several choice alternatives at the same time. In two applications, we have demonstrated the ability of our approach to produce accurate predictions while the extant model is prone to over-predictions (by 40%–80%). However, drivers of secondary demand and market share predictions seem largely unaffected by choice-set size variation.

If the main interest is to understand and predict market shares, discrete choice conjoint may be sufficient. However, when volume predictions are a key objective, volumetric conjoint can be a powerful tool—provided that the proposed set size adjustment specification is used.

The proposed model has implications for conjoint analysis, data fusion and modeling transaction data. All these applications can involve significant variation in choice-set size. Transaction data can include assortment changes over time, or varying assortment sizes in different stores. Data fusion involving choice experiments and transaction data is likely to involve dramatically different set sizes.

There are some limitations of our study: We only show applications to conjoint, but application to transaction data or a combination of transaction and choice experiment data could provide further evidence for the validity of the model. Transaction data would also allow study of stockpiling and purchase timing and may require attention to issues of endogeneity. It might be interesting to study the consequences of controlling for set size variation when modeling stockpiling or other endogeneity issues.

The model is implemented in the echoice2 package, which is available on github: https://github.com/ninohardt/echoice2

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